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THE $\Delta I = \frac{1}{2}$ RULE AND NON-PERTURBATIVE EFFECTS IN QUANTUM CHROMODYNAMICS

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ABSTRACT

Instanton-induced effects in QCD give rise to new, pure $\Delta I = \frac{1}{2}$ operators in the $|\Delta S| = 1$ weak effective Lagrangian. A new formalism is introduced such that the effects of instantons of up to a given size are explicitly taken into account, and such that a modified form of renormalization group equations hold. The results of our analysis suggest that instanton-induced effects might be important in the enhancement of the $\Delta I = \frac{1}{2}$ weak non-leptonic decay amplitudes.

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1. In the standard approach to the non-leptonic $|\Delta S| = 1$ decays, the weak effective Lagrangian is given by[1)-3)]

$$\mathcal{L}_{eff}(\Delta S=-1) = -\sqrt{2}\, G_F \cos\theta_c \sin\theta_c \sum_{i=1}^{6} C_i(\mu)\, O_i(x,\mu) \,, \tag{1}$$

where

$$O_1 = (1/2)\{(\bar{u}_L\gamma^\mu s_L)(\bar{d}_L\gamma_\mu u_L) - (\bar{u}_L \leftrightarrow \bar{d}_L)\} \,, \tag{2a}$$

$$\begin{aligned} O_2 = {} & (\bar{d}_L\gamma^\mu s_L)(\bar{u}_L\gamma_\mu u_L) + (\bar{u}_L\gamma^\mu s_L)(\bar{d}_L\gamma_\mu u_L) \\ & + 2(\bar{d}_L\gamma^\mu s_L)(\bar{d}_L\gamma_\mu d_L) + 2(\bar{d}_L\gamma^\mu s_L)(\bar{s}_L\gamma_\mu s_L) \,, \end{aligned} \tag{2b}$$

$$\begin{aligned} O_3 = {} & (\bar{d}_L\gamma^\mu s_L)(\bar{u}_L\gamma_\mu u_L) + (\bar{u}_L\gamma^\mu s_L)(\bar{d}_L\gamma_\mu u_L) \\ & + 2(\bar{d}_L\gamma^\mu s_L)(\bar{d}_L\gamma_\mu d_L) - 3(\bar{d}_L\gamma^\mu s_L)(\bar{s}_L\gamma_\mu s_L) \,, \end{aligned} \tag{2c}$$

$$O_4 = (\bar{d}_L\gamma^\mu s_L)(\bar{u}_L\gamma_\mu u_L) + (\bar{u}_L\gamma^\mu s_L)(\bar{d}_L\gamma_\mu u_L) - (\bar{d}_L\gamma^\mu s_L)(\bar{d}_L\gamma_\mu d_L) \,, \tag{2d}$$

$$O_5 = (\bar{d}_L\gamma^\mu \lambda^a s_L)(\bar{u}_R\gamma_\mu \lambda^a u_R + \bar{d}_R\gamma_\mu \lambda^a d_R + \bar{s}_R\gamma_\mu \lambda^a s_R) \,, \tag{2e}$$

$$O_6 = (\bar{d}_L\gamma^\mu s_L)(\bar{u}_R\gamma_\mu u_R + \bar{d}_R\gamma_\mu d_R + \bar{s}_R\gamma_\mu s_R) \,. \tag{2f}$$

Perturbative QCD effects at scales from $\mu \sim 1$ GeV to $\mu \sim M_W$ are taken into account in the Wilson coefficients C_i via the renormalization group[2)]. Unknown hadronic matrix elements of the operators O_i are usually estimated by means of PCAC, current algebra and factorization. These approximations do reflect one aspect of the non-perturbative dynamics of QCD: the spontaneous breakdown of the chiral symmetry due to quark condensation.

Unfortunately, the conventional approach is not capable of explaining the observed enhancement factor in the $\Delta I = \frac{1}{2}$, $|\Delta S| = 1$ non-leptonic decay amplitudes.

[Typically, one finds an enhancement of the $\Delta I = \frac{1}{2}$ part by a factor $4 \sim 5$ relative to the $\Delta I = 3/2$ part, to be compared to ~ 22 observed in the case of $K \to \pi\pi$ decays[1),3)].]

In this paper, we wish to consider the possible role of instanton-induced non-perturbative effects in QCD in the non-leptonic weak decays. We recall that the topologically non-trivial gauge-field configurations, such as instantons, are believed to be responsible for the abnormally large mass of η (as compared to m_π) and of η' (relative to m_π and m_K)[4)].

Our basic observation is that new operators which are all pure $\Delta I = \frac{1}{2}$ appear on the right-hand side of Eq. (1) as a result of instanton-induced corrections. Among the new operators, the one of lower dimension is

$$\begin{aligned} \tilde{O}_1 = (1/2)\{&(\bar{u}_R s_L)(\bar{d}_R u_L) - (1/4)(\bar{u}_R \sigma^{\mu\nu} s_L)(\bar{d}_R \sigma_{\mu\nu} u_L) \\ &- (3/4)(\bar{u}_R \lambda^a s_L)(\bar{d}_R \lambda^a u_L) + (3/16)(\bar{u}_R \lambda^a \sigma^{\mu\nu} s_L)(\bar{d}_R \lambda^a \sigma_{\mu\nu} u_L)\} \\ &- (\bar{u}_R \leftrightarrow \bar{d}_R) \;. \end{aligned} \tag{3}$$

Correspondingly,

$$\Delta \mathcal{L}_{eff} = -\sqrt{2}\, G_F \sin\theta_c \cos\theta_c \; \tilde{C}_1(\mu)\, \tilde{O}_1(x,\mu) \;. \tag{4}$$

The origin of this new effective interaction is simply understood by considering the amplitude shown in Fig. 1. Notice the non-trivial axial U(1) properties of $\tilde{O}_1$ characteristic of instanton amplitudes. The Lorentz and colour structure of $\tilde{O}_1$ can be (conveniently) obtained by a simple convolution of the operator O_1, Eq. (2a), with the effective four-fermion interaction Lagrangian due to instantons[5)*)].

A crucial observation is that only the operator O_1, which is pure $\Delta I = \frac{1}{2}$, gets instanton corrections. This is due to the SU(2) × SU(2) singlet nature of instanton effective interactions. More generally, all instanton-induced interactions are completely antisymmetric in the left-handed quark flavours (as well as in the outgoing right-handed flavours). On the other hand, the operators O_2, O_3

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*) The correctness of such a procedure can be checked by computing a free-quark matrix element corresponding to Fig. 1, by use of the LSZ formulae in the presence of an instanton (in the singular gauge). It is such a calculation that leads to the result in Eq. (10). Details will appear elsewhere.

and O_4 (we ignore O_5 and O_6 which have small coefficients) are all symmetric with respect to the flavours of the outgoing left-handed quarks. Thus instanton corrections affect O_1 only, a pure $\Delta I = \frac{1}{2}$ and SU(3) octet operator, and give rise to a new operator $\tilde{O}_1$, with the same selection rules.

Analogously, instanton-induced corrections give rise to a new dimension-nine, pure $\Delta I = \frac{1}{2}$ operator as well,

$$\Delta \mathcal{L}'_{eff} = -\sqrt{2}\, G_F \sin\theta_c \cos\theta_c \; c_9 \, O_9 \quad , \tag{5}$$

where (see Fig. 2),

$$O_9 = (1/6) \sum_{P(\bar{u}_R, \bar{d}_R, \bar{s}_R)} (-)^P \left[(\bar{d}_R d_L)(\bar{s}_R \sigma_{\mu\nu} u_L)(\bar{u}_R \sigma^{\mu\nu} d_L) + \ldots \right] \tag{6}$$

2. - It might be thought that the instanton effects can be taken into account in the matrix elements of Eq. (1). Actually, there are two questions to be answered: whether the operator product expansion involved in the derivation of Eq. (1) gets modified by small ($\lesssim 1/M_W$) instantons, and how various operators "normalized at μ" are to be defined in the presence of instantons of all sizes. As for the first point, there is convincing evidence[6),7)] that instantons in certain cases do modify the standard operator product expansion by adding new operators on the right-hand side and/or by introducing non-perturbative coefficients. In our case, too, a close look at the instanton contribution shows that Eq. (1) does not represent a correct answer, even at $\mu = M_W$ (the details will appear elsewhere). However, the correction term, Eq. (4), has a tiny coefficient (for $\mu = M_W$), proportional to $\exp\{-2\pi/\alpha_s(M_W)\}$, and thus will not be of practical importance in our discussion of non-leptonic decays.

The second question is a more relevant one. It is true that the structure of ultra-violet divergences of the theory is unaffected by the presence of instantons. [This can be checked, for example, by using our result Eq. (13) below: the evolution matrix $P \exp \int dt' \Gamma$ can be rewritten as a product of a diagonal matrix involving the standard log M_W factors, and a non-diagonal matrix which is finite in the limit $M_W \to \infty$.] The standard approach, in which the μ-dependence of various operators arises only through the ultra-violet divergences they cause upon insertion, however, leads to a practical difficulty. When the relevant hadronic matrix elements are such that large instantons [of sizes up to $\rho \sim O(1/\Lambda_{QCD})$] contribute, as in our problem of non-leptonic decays, one must face the problem of the infra-red: confinement, necessity of going beyond the one-instanton approximation, etc. In any case, in this standard approach, all the effects of instantons being ultra-violet finite will be included in the unknown matrix elements; such an approach will not be useful for the purpose of estimating instanton effects.

Actually, the contribution of not-too-large instantons is well defined and computable. Indeed, instantons of a given size ρ affect importantly the relevant operator, say O_1 (e.g., change their chirality), at distances of order ρ, so that in their presence only the quantum fluctuations with wavelengths less than ρ are described by the standard (perturbative) Wilson coefficient C_1. On the other hand, the fluctuations of larger wavelengths ($\gg \rho$) are described by the anomalous dimension of the new effective operator $\tilde{O}_1$ (see Fig. 1).

Such a physical picture suggests the following procedure which allows us to take account of the effects associated with instantons up to a fixed size $1/\mu$, which can be chosen at our will. By choosing μ small enough (but still not so small as to invalidate completely the one-instanton approximation), we obtain a semiquantitative estimation of the instanton effects.

3. To simplify our argument, from here on we shall focus on the contribution of $\Delta\mathcal{L}_{eff}$, Eq. (4) (namely, one-instanton effects proportional to m_s). First, the standard OPE result, Eq. (1), is generalized to include the $\tilde{C}_1\tilde{O}_1$ term of Eq. (4). Second, we define, with Novikov and others[6)], the operator matrix elements $\langle O(\mu)\rangle$ to include effects associated with instantons of sizes $\rho > 1/\mu$[*)]. Such a splitting of the integration over instanton size is always possible, whatever renormalization conditions for the operators, normalized (for convenience) at the same μ, are chosen.

Renormalizability of the theory implies the equation (we consider only the sector involving O_1 and $\tilde{O}_1$)

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*) This definition corresponds to the effective operators à la Wilson: the high-frequency fluctuations above μ (perturbative or non-perturbative) are integrated out, and the resulting operators are treated as local operators as far as the physics at long wavelengths ($> 1/\mu$) is concerned. Such a definition is precisely what is needed if one wishes to apply our results to lattice calculations.

$$(\mathcal{D} - \Gamma + \ldots)\begin{pmatrix} \langle T\{O_1 \cdots\}\rangle \\ \langle T\{\tilde{O}_1 \cdots\}\rangle \end{pmatrix} = 0 \quad , \tag{7}$$

$$\mathcal{D} = \mu\frac{\partial}{\partial\mu} + \beta(g,\lambda)\frac{\partial}{\partial g} + \beta_\lambda(g,\lambda)\frac{\partial}{\partial\lambda} \quad ,$$
$$\lambda \equiv m_s/\mu \tag{8}$$

where an arbitrary set of local fields and their anomalous dimensions are left implicit in Eq. (7). The matrix Γ defined by Eq. (7) is of the form

$$\Gamma = \begin{pmatrix} \gamma_0 & \gamma_{-1} \\ \gamma_1 & \tilde{\gamma}_0 \end{pmatrix} ,$$

$$\gamma_0 = \frac{2}{\pi}\alpha_s + \ldots \qquad (\alpha_s \equiv g^2/4\pi) \quad . \tag{9}$$

γ_0 is the standard anomalous dimension of the operator O_1, and γ_1, due to an instanton (of topological charge 1) can be computed from Fig. 1, to first order in λ,

$$\gamma_1 = c_0\,\lambda\left(\frac{2\pi}{\alpha_s}\right)^6 e^{-2\pi/\alpha_s} \quad . \tag{10}$$

c_0 is a numerical constant which depends on the definition of the QCD coupling constant α_s. In the $\overline{\mathrm{MS}}$ scheme,

$$c_0 \simeq 5.4 \times 10^{-3} \quad . \tag{11}$$

$\gamma_{-1} \approx 9.0\ \gamma_1$ is due to an anti-instanton which gives back O_1 from $\tilde{O}_1$, and $\tilde{\gamma}_0$ is the perturbative anomalous dimension of $\tilde{O}_1$ (which is not needed below).

Since the physics is independent of μ, one finds

$$(\mathcal{D} + \Gamma)\begin{pmatrix} C_1 \\ \tilde{C}_1 \end{pmatrix} = 0 \quad . \tag{12}$$

The solution of Eq. (12) is

$$\begin{pmatrix} C_1 \\ \tilde{C}_1 \end{pmatrix}_{\mu, g, \lambda} = \left[\mathcal{P} \exp \frac{1}{2} \int_0^t dt' \, \Gamma\left(\bar{g}(t', g, \lambda), \bar{\lambda}(t', g, \lambda)\right) \right] \begin{pmatrix} C_1 \\ \tilde{C}_1 \end{pmatrix}_{\substack{\mu = M_W , \\ g = \bar{g}(M_W) , \\ \lambda = \bar{\lambda}(M_W)}} \tag{13}$$

where $t = \log M_W^2/\mu^2$ and $(C_1, \tilde{C}_1) \simeq (1,0)$ at $\mu = M_W$ *).

Exact evaluation of Eq. (13) is difficult, but since γ_1 and γ_{-1} are significant only at small μ (large t), Eq. (13) may be calculated approximately. One finds ($1/b_0\pi \simeq 0.48$ for $N_f = 4$, 0.44 for $N_f = 3$)

$$C_1(\mu) \simeq \exp \frac{1}{2} \int_0^t \gamma_0 \, dt' = \left[\alpha_s(\mu)/\alpha_s(M_W) \right]^{1/b_0\pi} \quad , \tag{14a}$$

$$\tilde{C}_1(\mu) \simeq C_0 m_s \int_{1/M_W}^{1/\mu} d\rho \left(\frac{2\pi}{\alpha_s(1/\rho)} \right)^6 e^{-2\pi/\alpha_s(1/\rho)} \cdot \left\{ \alpha_s(1/\rho)/\alpha_s(M_W) \right\}^{1/b_0\pi}$$

$$\simeq 1.3 \cdot 10^{-3} \left(\frac{m_s}{\mu} \right) \left(\frac{2\pi}{\alpha_s(\mu)} \right)^6 e^{-2\pi/\alpha_s(\mu)} \quad . \quad \left(\alpha_s(\mu) \lesssim 1 \right) \tag{14b}$$

In Eq. (14b), the sign of $\tilde{C}_1$ seems to depend on the sign of m_s, which is in turn normally conventional. As a matter of fact, the sign of $\tilde{C}_1$ is fixed to be

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*) Equation (13) clarifies the meaning of the approximation involved. Namely, the momentum space between the scales μ and M_W is divided into ordered shells, and in each of them either the perturbative (γ_0, $\tilde{\gamma}_0$) or non-perturbative (γ_1, γ_{-1}) effects are computed. The sum is then made over all possible ways in which the momentum space is divided, and over the choices in each shell. It is a generalization of the strong-ordering approximation, used to get the leading perturbative corrections.

positive by the following argument. For an arbitrary θ, the QCD vacuum parameter, m_s should be replaced in Eq. (14b) by $m_s \exp(i\theta)$. Now the absence of strong CP violation and further considerations of the pseudoscalar mass spectrum imply that $\theta \simeq 0$ [8),9)] (with a definition of the quark fields that makes all quark masses real and positive).

4. It can be seen from Eq. (14b) that the coefficient $\tilde{C}_1(\mu)$ is small unless $\mu \lesssim 1$ GeV. However, at μ such that $\alpha_{\overline{MS}}(\mu) \sim 1$, we find*)

$$\tilde{C}_1(\mu) \simeq 0.15\, m_s/\mu \tag{15}$$

Notice that this estimate is (for $\mu \sim m_s$) not smaller than the corresponding coefficient of the penguin operator O_5, $|C_5| \simeq 0.04 \sim 0.1$.

The fact that the chiral properties of the new operator $\tilde{O}_1$, involving the right-handed quarks, are somewhat similar to O_5, suggests a comparable enhancement of its matrix elements as compared to those of $O_1 \sim O_4$. We conclude that the new, instanton-induced effects are at least as important as the so-called penguin contribution.

So far we have concentrated our interest on $\Delta\mathcal{L}_{eff}$ of Eq. (4). A complete analysis requires an analogous study of $\Delta\mathcal{L}'_{eff}$ of Eq. (5), where the structure of the renormalization group equations is more complicated, more operators having to be introduced. A rough estimate of C_9 gives an expression similar to Eq. (14b), with the substitution $m_s/\mu \to 1/\mu^3$ and with a comparable coefficient. O_9 is also a pure $\Delta I = \frac{1}{2}$ operator.

Altogether, we believe that the new, instanton-induced effects deserve careful attention. At the moment, we are unable to make more definite statements, because of the lack of a reliable estimation of the various hadronic matrix elements (old and new ones). There are also some uncertainties as to the instanton density itself in the physical vacuum which, however, work in the direction

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*) The (one-instanton) approximation Eq. (14b) is not quite adequate for $\alpha(\mu)$ not sufficiently small. Actually, a more refined estimation of Eq. (13) by including the sum over arbitrary numbers of instantons and anti-instantons gives $\tilde{C}(\mu) \simeq 0.32\, m_s/\mu$ for $\alpha_s(\mu) \simeq 1$.

(if any) of enhancing the instanton effects[10].

Our conjecture that the instanton-induced effects are indeed responsible for the missing $\Delta I = \frac{1}{2}$ enhancement factor can, however, be tested (for instance, by using lattice QCD). If it were possible to compute the matrix elements of the standard weak effective Lagrangian, Eq. (1), for sufficiently large μ, one could safely neglect the new terms $\Delta\mathcal{L}_{eff}$ and $\Delta\mathcal{L}'_{eff}$. In this case, the bulk of the instanton-induced effects would be included in the matrix elements of the standard operator O_1. These matrix elements would therefore be enhanced, not only relative to those of the $\Delta I = 3/2$ operator O_4, but also compared to those of other $\Delta I = \frac{1}{2}$ operators O_2 and O_3 (and in particular, as compared to naive free-quark-model estimates). Furthermore, when going to lower values of μ, the inclusion of the new terms $\Delta\mathcal{L}_{eff}$ and $\Delta\mathcal{L}'_{eff}$ would be essential in order to get the μ-independence of the physical results.

Let us conclude with a few remarks. We have already noted that, as a trivial consequence of the flavour antisymmetrization associated with instanton-induced effects, all new contributions have the right properties with respect to isospin, $SU(2) \times SU(2)$[11], and SU(3) (an octet)[12]. Therefore all earlier successful predictions based on these symmetries remain intact. Also, we recall the well-known interconnection between the $\Delta I = \frac{1}{2}$ problem and the CP-violation parameter ratio ε'/ε[13]. Since the instanton-induced corrections do not involve heavy flavours (t and b quarks), their magnitude is unrelated to the extent of CP violation in the kaon decays in the standard model with the Kobayashi-Maskawa quark mixing. Thus the presumed dominance of instanton effects in these decays is perfectly compatible with a small observed ratio ε'/ε[14].

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FIGURE CAPTIONS

Fig. 1 : The weak amplitude for the transition $s_L u_L \to u_R d_R$ in the presence of an instanton (shown by a shaded disc). Dominant effects occur at distances $|z-x| \sim \rho$.

Fig. 2 : Instanton-induced transition, $s_R u_R d_R \to u_L d_L d_L$ or $s_L u_L s_L \to u_R d_R s_R$.

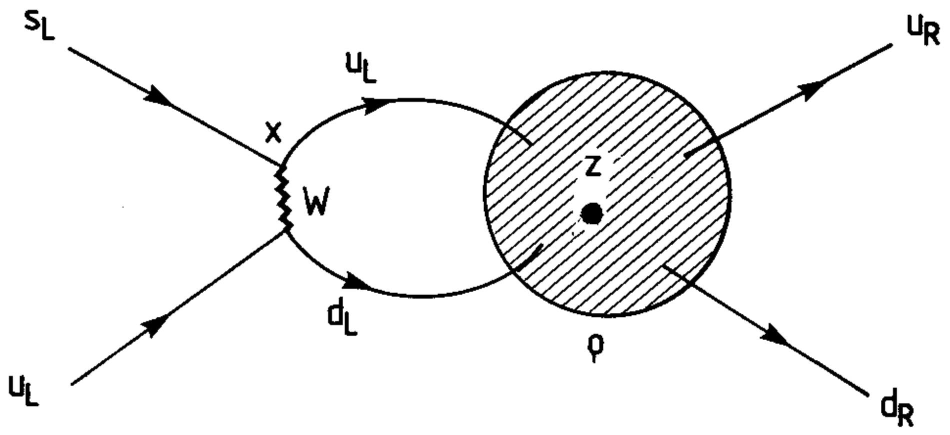

FIG. 1

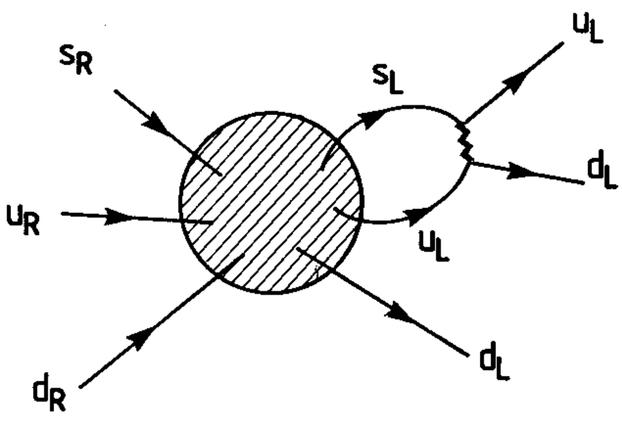

FIG. 2a

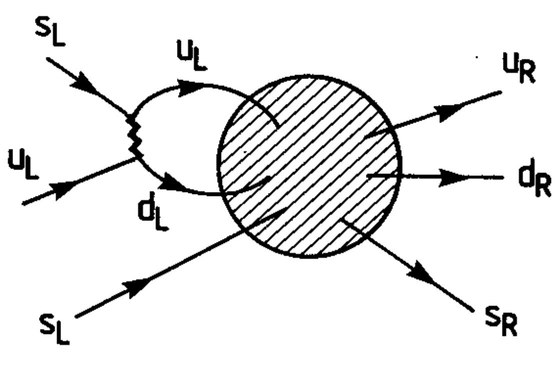

FIG. 2b