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QUARK MASSES AND MIXING ANGLES FROM UNIVERSAL SEESAW MECHANISM

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Abstract

A universal Seesaw Mechanism is invoked to account for the observed fermion mass hierarchies. In the framework of left-right symmetry, heavy fermions (mass scale χ), which are $SU(2)_L \otimes SU(2)_R$ singlets are postulated while retaining the simplest possible Higgs system, namely $\varphi(1,2,1)_{-1} \oplus \varphi(1,1,2)_{-1}$ accompanied by a left-right singlet, $\sigma(1,1,1)_0$, in the standard $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \times U(1)_{B-L}$ notation. Every conventional quark and lepton is accompanied by a non-mirror singlet heavy fermion, so that the associated mass matrix is doubled and has the seesaw form usually associated only with the neutrino mass matrix. In the single generation case, the model provides a plausible explanation for the mass hierarchy $m_{e,u,d} \sim 10^{-4} M_W$ and predicts $m_{\nu_e} m_{\nu_R} \approx m_e^2$, thus accounting for the superlightness of neutrinos. Combined with an U(1) axial symmetry, the mechanism provides a formalism in which the generations are distinguished and constraints emerge on the allowed form of mass matrices. In this paper, we consider the realistic case of three generations in a simplified version of the model in which CP violation does not arise from the gauge sector. Choosing the $U(1)_A$ quantum numbers so that the mass matrices are of Fritzsch type, we calculate experimentally measured Cabibbo-Kobayashi-Maskawa matrix elements, V_{us}, V_{ub}, and V_{cs} and derive their dependence on quark mass parameters. An interesting correlation between V_{us} which measures the Cabibbo angle and V_{ub} which measures the charmless decay of the b-quark emerges from the model. V_{ub} is naturally suppressed if $V_{us} = (\sqrt{\frac{\bar{d}}{s}} - \sqrt{\frac{\bar{u}}{s}})$ to a very good approximation.

I. INTRODUCTION

The situation regarding the standard theory of electroweak interactions (Glashow-Salam-Weinberg) is reminiscent of the years when quantum electrodynamics (QED) was put to more and more rigorous experimental tests with the hope of finding serious discrepancies with the theoretical predictions. But QED worked extremely well and its theoretical shortcomings - the infinities, the e-μ puzzle, the incalculability of the masses and coupling constants of the theory - have remained unresolved. The standard GSW theory suffers from similar shortcomings and more. In spite of its remarkable empirical successes, the theory remains unsatisfactory for reasons so well articulated in the literature that it does not warrant repetition. The generation puzzle and fermion mass hierarchies are two of several puzzles which have attracted a great deal of attention in recent years.

The e-μ puzzle by itself was more of a theoretical and of an academic nature. It had few experimental consequences outside the domain of QED. In contrast, the generation puzzle presents itself in a much more extended form with consequences covering a wide range of new experimental findings associated with the physics of the new flavors. For instance, the generalized Cabibbo-type angles and phases (Kobayashi-Maskawa (KM) angles and phases), which come into play when one transforms the gauge eigenstates into mass eigenstates, govern the charm, bottom, and top quark decays, weak CP-violation, and yield quantitative estimates of the rare decay modes. The number of flavors, especially the number of light neutrinos and their masses, have important implications in astrophysics and cosmology. Thus generally speaking, a whole range of phenomena depend on our understanding of the generation puzzle, the intra-generational interactions, the form of the mass matrices, and the symmetry breaking mechanism. The standard theory has no answer to these questions and hence it is necessary to go beyond its framework.

Recent attempts in this direction have covered a wide range of ideas incorporating continuous and discrete symmetries,[1] radiative mechanisms for mass generation,[2] seesaw mechanisms,[3] and so on. The purpose of our paper is to explore in some detail the idea of a universal seesaw mechanism (USM) proposed by two of the current authors.[4,5] The mechanism is introduced within the framework of left-right symmetric $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ model. The most important departure from the conventional left-right symmetric models is the extension of the fermionic sector to include a set of new fermions[6] which are singlets under $SU(2)_L \otimes SU(2)_R$. Every conventional quark and lepton is accompanied by a new singlet partner. Also unlike in the conventional left-right symmetric models, the Higgs system is kept minimal in that it is the straightforward generalization of the single Higgs doublet in the standard theory to left-right symmetric combination of two Higgs doublets. The consequence of this simple idea is that the tree level mass matrix for every fermion has the seesaw form of a $2\otimes 2$ matrix, strictly analogous to that introduced to explain the superlightness of the neutrino.[3]

In the next section, we review briefly the case of a single generation. We show that the most general potential consisting of the minimal Higgs system coupled with a singlet Higgs scalar whose predominant role is to break the left-right symmetry allows in a certain range of parameters an absolute minimum of the potential, which will be consistent with a hierarchy in the vacuum expectation values (VEVs) necessary to explain the mass hierarchies in the fermionic system. In the quark and the charged leptonic sectors, the seesaw form of the mass matrices and the assumed hierarchy in the VEVs establishes the smallness of the masses of the u and d quarks and the electron. It then follows without any further assumptions a formula for the neutrino masses, namely,

$$m_{\nu_L} m_{\nu_R} \approx m_e^2,$$

which correlates the smallness of the neutrino mass with the smallness of the electron mass.

In Section III, we discuss the general formalism for N generations. By imposing a global axial symmetry U(1), we distinguish the generations and show how the U(1) quantum numbers restrict the form of the mass matrices. The assumption of this extra global symmetry necessitates the doubling of the left- and right- doublet Higgs fields. We then have a seesaw form for all the fermionic mass matrices. Two cases arise: a) the mass matrix of the extra fermions is non-singular, in which case an effective $N \otimes N$ mass matrix can be written for the conventional light quarks; b) the relevant mass matrix of the heavy fermions is singular. The latter instance has no simplifying feature and gives rise to a qualitatively different mass spectrum. In Section IV, we apply these considerations to the realistic case of three generations with prescribed quantum numbers for the generations so that all the sub-mass-matrices are Fritzsch type. By assuming strict equality of the VEVs of the two left and the two right Higgs scalars separately, but the left VEV being very much smaller than the right VEV, we simplify the model. The effective light quark mass matrix is then real and symmetric. The expressions for the mixing angles involve only the quark masses and two scale parameters. A much desired formula for the well-known Cabibbo angle emerges in terms of the quark masses; also the other two matrix elements (V_{cb} and V_{ub}) in the Cabibbo-Kobayashi-Maskawa (CKM) matrix can be easily made consistent with the known experimental information concerning B-meson life-time and also the known limits on its charmless decays. In fact, an extremely interesting correlation between the expression for the Cabibbo angle (the matrix element V_{us}) and V_{ub} explains naturally the suppression of the latter. Of course, by allowing the VEVs to be complex, we can introduce CP violation with a nonvanishing CP phase while retaining the good features of the results for mixing angles. We also consider in this section the singular case and discuss its implications on the mass spectrum. The final section is devoted to a summary and discussion of the results. In a follow up paper, we propose to extend these considerations to leptons with particular interest to neutrinos, and also CP violation.

II. SINGLE GENERATION CASE

(a) General Considerations

Consider the standard model of electro/weak and strong interactions based on the group structure $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. Within the framework of this symmetry, the familiar quarks and leptons belong to the complex representation

$$q_L(3,2,1)_{1/3}\ ,\ q_R(3,1,2)_{1/3},$$
$$\ell_L(1,2,1)_{-1}\ ,\ \ell_R(1,1,2)_{-1}, \tag{2.1}$$

where $(a,b,c)_d$ label respectively $SU(3)_C$, $SU(2)_L$, $SU(2)_R$, representations and d, the quantum number Y=B–L of $U(1)_{B-L}$. In addition to the above conventional fermions, we postulate new fermions which belong to a real representation.[6] They share identical $SU(3)_C$ quantum numbers with the conventional fermions, but differ from them since they are assumed to be $SU(2)_L \otimes SU(2)_R$ singlets. We denote them by

$$U_{L,R}(3,1,1)_{4/3},\quad D_{L,R}(3,1,1)_{-2/3},$$

$$N_{L,R}(1,1,1)_0 \quad , \quad E_{L,R}(1,1,1)_{-2}. \tag{2.2}$$

The Y=B–L quantum numbers of the new fermions are chosen so that every ordinary fermion has a $SU(2)_L \otimes SU(2)_R$–singlet companion with identical $SU(3)_C \times U(1)_Q$ assignments ($Q=T_{3L}+T_{3R}+\frac{1}{2}(B-L)$). The associated primary Higgs system to which the fermions couple is assumed to be

$$\varphi_L(1,2,1)_{+1} + \varphi_R(1,1,2)_{+1}, \tag{2.3}$$

which provides a minimal Higgs system and a natural left–right symmetric generalization of the single Higgs doublet of the Glashow–Salam–Weinberg $SU(2)_L \times U(1)$ electro/weak theory. We shall not assume at the start the conventional sources of quark/lepton masses in left–right symmetric models, namely,

$$\varphi(1,2,2)_0 + \varphi(1,3,1)_{-2} + \varphi(1,1,3)_{-2}. \tag{2.4}$$

The above scalars can be looked upon as bilinears of our primary system (2.3), as <u>effective scalar fields</u> in the low–energy limit.[4]

However, with only (2.3) present, left–right symmetry remains unbroken. To obtain a desired tree–level hierarchy $v_L \ll v_R$, where $\langle\varphi_{L,R}\rangle = v_{L,R}$, we have to break L–R symmetry <u>explicitly</u>. In reference 4, we had assumed that such explicit L–R symmetry breaking is the consequence of physics beyond $SU(3)_C \otimes SU(2)_L \otimes U(1)_{B-L}$ where it is broken only spontaneously. This was a realistic possibility that emerged from our $SU(5)_L \otimes SU(5)_R$ grand unification scheme;[7] $SU(2)_L$ and $SU(2)_R$ are not allowed to share a common evolution in that scheme if $\sin^2\theta_W$ were to conform to experimental value with three or four generations of quarks and leptons.

There is, however, an elegant way of breaking the L–R symmetry spontaneously that was first considered by Chang, Mahopatra and Parida.[8] The introduction of a scalar singlet $\sigma(1,1,1)_0$ with odd under $L\leftrightarrow R$, even under charge conjugation, and hence odd under CP, allows in the Higgs potential, a term which can break L–R symmetry spontaneously. Let χ denote the vacuum expectation value of this singlet scalar, namely, $\chi=\langle\sigma(1,1,1)_0\rangle$. We would like to postulate the hierarchy $v_L \ll v_R \ll \chi$. Assuming for the moment that we can construct a Higgs potential involving φ_L, φ_R, and σ, such that its absolute minimum allows the desired hierarchy, let us consider the quark–Higgs sector with the couplings

$$\begin{aligned}\mathcal{L}_Y = \; & (Y_d \bar{q}_L \varphi_L D_R + Y_u \bar{q}_L \tilde{\varphi}_L U_R + (L\to R)) + \text{H.C.} \\ & + (Y_U \bar{U}_L U_R + Y_D \bar{D}_L D_R)\sigma + \text{H.C.}\end{aligned} \tag{2.5}$$

In (2.5), $\tilde{\varphi}_L = i\tau_2 \varphi_L^*$. When σ acquires a nonvanishing vacuum expectation value χ, the singlet quarks acquire a mass of the order χ. Y_d, Y_u, Y_U, Y_D are Yukawa–type coupling constants[9] which we can assume to be real without any loss of generality.

Let

$$\begin{aligned}\langle Y_d \varphi_L\rangle &= Y_d v_L = V_L^d; \quad \langle Y_d \varphi_R\rangle = Y_d v_R = V_R^d \\ \langle Y_D \sigma\rangle &= Y_D \chi = \chi^d\end{aligned} \tag{2.6}$$

Then, once the gauge symmetry is spontaneously broken down to $SU(3)_C \times U(1)_Q$, the tree–level mass matrix, in the down sector for instance, is given by

$$M_d = \begin{bmatrix} 0 & V_L^d \\ V_R^d & \chi^d \end{bmatrix} . \tag{2.7}$$

The corresponding tree–level mass matrix in the up–sector is

$$M_u = \begin{bmatrix} 0 & V_L^u \\ V_R^u & \chi^u \end{bmatrix} , \tag{2.8}$$

where

$$V_L^u = Y_u v_L^* = \frac{Y_u}{Y_d} V_L^{d*}, \quad V_R^u = Y_u v_R^* = \frac{Y_u}{Y_d} V_R^{d*}, \quad \chi^u = Y_u \chi = \frac{Y_U}{Y_D} \chi^d. \tag{2.9}$$

Thus we have a generic seesaw form for the mass matrix in both the charge sectors of the quarks.

In a similar fashion, we can write down the lepton-Higgs interactions in the lepton sector with $E_{L,R}$ and $N_{L,R}$ as corresponding singlet partners of the $e_{L,R}$ and $\nu_{L,R}$. We have

$$\begin{aligned}\mathscr{L}_L = &(Y_e \bar{l}_L \varphi_L E_R + Y_{\nu_L} \bar{l}_L \tilde{\varphi}_L N_R + L \leftrightarrow R) + \text{H.C.} \\ &+ (Y'_{\nu_L} \ell_L^T C \tilde{\varphi}_L N_L + Y'_{\nu_R} \ell_R^T C \tilde{\varphi}_R N_R) + \text{H.C.}\end{aligned} \tag{2.10}$$

Note that in (2.10), the first line involves couplings which are in exact analogy with the quark sector. The terms in the second line will give rise to Majorana mass terms, so that for the charged leptons, we have the mass matrix

$$M_e = \begin{bmatrix} 0 & V_L^e \\ V_R^e & \chi^e \end{bmatrix}, \tag{2.11}$$

whereas for the neutrino (in the space, (ν_L, ν_R, N_L, N_R)), we have the mass matrix[4]

$$M_\nu = \begin{bmatrix} 0 & 0 & Y_{\nu_L} v_L & Y'_{\nu_L} v_L \\ 0 & 0 & Y_{\nu_R} v_R^* & Y'_{\nu_R} v_R^* \\ Y_{\nu_L} v_L & Y_{\nu_R} v_R^* & \chi_1 & \chi \\ Y'_{\nu_L} v_L & Y'_{\nu_R} v_R^* & \chi & \chi_2 \end{bmatrix}. \tag{2.12}$$

b) Phenomenology of the Single Generation Model

From (2.7), (2.8), (2.11), and (2.12) (with $\chi_1 \sim \chi_2 \sim \chi$), we identify the masses of d and u quarks, electron, and neutrinos as given by

$$m_u = \frac{v_L^d v_R^d}{\chi} = (Y_d)^2 \frac{v_L v_R}{\chi}, \quad m_d = \frac{v_L^u v_R^u}{\chi} = (Y_u)^2 \frac{v_L v_R}{\chi}, \quad m_e = \frac{v_L^e v_R^e}{\chi} = (Y_e)^2 \frac{v_L v_R}{\chi}, \tag{2.13}$$

and

$$m_{\nu_L} = \frac{Y_{\nu_L} Y'_{\nu_L} v_L^2}{\chi} \quad , \quad m_{\nu_R} = \frac{Y_{\nu_R} Y'_{\nu_R} v_R^2}{\chi}. \tag{2.14}$$

Clearly, if all the Yukawa couplings are equal,

$$m_u = m_d = m_e \tag{2.15}$$

and

$$m_{\nu_L} m_{\nu_R} = m_e^2. \tag{2.16}$$

However, it would be interesting to see whether we can obtain agreement with the known or expected values of quark and lepton masses <u>without</u> introducing additional hierarchies in the Y-couplings. We have

$$\frac{m_u}{m_d} = \left[\frac{Y_u}{Y_d}\right]^2 \simeq \frac{1}{2}, \tag{2.17}$$

which implies $Y_u = \frac{1}{\sqrt{2}} Y_d$, and hence Y_u, Y_d are of the same order of magnitude.

Let us further note, assuming the gauge couplings $g_L = g_R = g$, that

$$M_{W_L} = \tfrac{1}{2} g v_L \text{ and } M_{W_R} = \tfrac{1}{2} g v_R. \tag{2.18}$$

Let us also assume, based on the current experimental information,

$$M_{\nu_L} < 18 \text{ eV} \tag{2.19}$$

and

$$10^8 \text{ GeV} < \chi < 10^{12} \text{ GeV}. \tag{2.20}$$

The above limit follows from astrophysical and cosmological considerations,[10] provided we identify[5] σ as the singlet scalar that breaks Peccei–Quinn, global, axial $U(1)_{PQ}$ symmetry as a device to avoid the strong CP violation.[11]

Then, in the case of leptons, we have from (2.13) and (2.14),

$$\frac{m_{\nu_L}}{m_e} = \frac{Y_{\nu_L} Y'_{\nu_L}}{Y_e^2} \left(\frac{v_L}{v_R}\right), \quad \frac{m_{\nu_R}}{m_e} = \frac{Y_{\nu_R} Y'_{\nu_R}}{Y_e^2} \left(\frac{v_R}{v_L}\right). \tag{2.21}$$

Hence, if $\frac{Y_{\nu_L} Y'_{\nu_L}}{Ye^2}$ and $\frac{Y_{\nu_R} Y'_{\nu_R}}{Ye^2}$ are of $O(1)$ as in the case of quark sector,

$$\frac{v_L}{v_R} \approx \frac{m_{\nu_L}}{m_e} \leq 3.5\text{x}10^{-5} \tag{2.22}$$

and therefore, $v_R > \frac{v_R}{3.5\text{x}10^{-5}} \approx O(10^7)$ GeV, where we have used $v_L = (\sqrt{2}G_F)^{-1/2} = 246$ GeV. From (2.18), then

$$M_{W_R} > \frac{M_{W_L}}{3.5\text{x}10^{-5}} \approx 2.3\text{x}10^6 \text{ GeV} \tag{2.23}$$

and from (2.21),

$$m_{\nu_R} > 14.5 \text{ GeV}. \tag{2.24}$$

As a final remark regarding these numbers, we note that if the Y-coupling ratios in (2.21) are $O(1)$, $\chi \approx v_L^2/m_{\nu_L} \approx (10^{12})$ GeV consistent with (2.20).

c) Spontaneous Symmetry Breaking

With the Higgs fields $\varphi_L(1,2,1)_{-1}$, $\varphi_R(1,1,2)_{-1}$, and $\sigma(1,1,1)_0$, the most general Higgs potential takes the form

$$V = \Lambda[(\varphi_L^+\varphi_L)^2+(\varphi_R^+\varphi_R)^2] - 2M^2[\varphi_L^+\varphi_L+\varphi_R^+\varphi_R]$$
$$+ 2\Gamma(\varphi_L^+\varphi_L)(\varphi_R^+\varphi_R) + \gamma\sigma^4-2m^2\sigma^2+2\gamma\sigma(\varphi_L^+\varphi_L-\varphi_R^+\varphi_R), \tag{2.25}$$

where Λ, M, Γ, λ, m, and γ are constants. Under the assumption $\sigma\leftrightarrow -\sigma$ when $L\leftrightarrow R$, V is L–R symmetric. Our object is to verify whether (2.25) allows, for a range of parameters $\Lambda,\ldots,\gamma$, the absolute minimum of V such that the desired hierarchy $v_L<<v_R<<\chi$ is attainable. First note the positivity conditions as σ, φ_L, and $\varphi_R\to\infty$,

$$\lambda>0, \ \Lambda>0, \ \Lambda+\Gamma>0 \tag{2.26}$$

Varying with respect to φ_L, φ_R, and σ, the condition for minimum of V gives

$$v_L(\Lambda v_L^2+\Gamma v_R^2 - M^2+\gamma\chi) = 0, \tag{2.27}$$

$$v_R(\Lambda v_R^2+\Gamma v_L^2 - M^2 - \gamma\chi) = 0, \tag{2.28}$$

$$\lambda\chi^3-m^2\chi + \frac{\gamma}{2}(v_L^2-v_R^2) = 0. \tag{2.29}$$

There are four cases to be studied; (i) $v_L=v_R=\chi=0$, (ii) $v_L=v_R=0$; $\chi\neq0$, (iii) $v_L=0$, $v_R\neq0$ and $v_L\neq0$, $v_R=0$, (iv) $v_L\neq0$, $v_R\neq0$. From (2.23)–(2.25), one gets,

$$\text{case (i):} \quad V_{min}^{(1)} = 0, \tag{2.30}$$

$$\text{case (ii):} \quad \chi^2= \frac{m^2}{\lambda}, \quad V_{min}^{(2)}(v_L=v_R=0;\ \chi= \pm\sqrt{\frac{m^2}{\lambda}}) = -\frac{m^4}{\lambda}, \tag{2.31}$$

so that for $m\neq0$, $V_{min}^{(2)} < V_{min}^{(1)}$.

$$\text{Case (iii):} \quad \Lambda v_R^2 = M^2+\gamma\chi, \tag{2.32}$$

where χ satisfies the cubic equation

$$\lambda\chi^3-(m^2+ \frac{\gamma^2}{2\Lambda})\chi - \frac{\gamma M^2}{2\Lambda} = 0. \tag{2.33}$$

In terms of χ, $V_{min}^{(3)}(v_L=0,\ v_R\neq0,\ \chi\neq0)=V_{min}^{(3)}(v_L=0)$ is given by

$$V^{(3)}_{min}(v_L=0)=-(m^2+\frac{\gamma^2}{2\Lambda})\chi^2-\frac{3}{2}\frac{M^2\gamma}{\Lambda}\chi-\frac{M^4}{\Lambda}. \tag{2.34}$$

Similarly, in the case of $v_L\neq0$, $v_R=0$, $\chi\neq0$,

$$V^{(3)}_{min}(v_R=0) = -(m^2+\frac{\gamma^2}{2\Lambda})\chi^2+\frac{3}{2}\frac{M^2\gamma}{\Lambda}\chi-\frac{M^4}{\Lambda}, \tag{2.35}$$

where we have used

$$\Lambda v_L^2 = M^2-\gamma\chi, \tag{2.36}$$

with χ satisfying a cubic equation analogous to (2.33). Finally,

$$\text{Case (iv):}\quad v_L^2=\frac{M^2}{\Lambda+\Gamma}-\frac{\gamma\chi}{\Lambda-\Gamma},\quad v_R^2=\frac{M^2}{\Lambda+\Gamma}+\frac{\gamma\chi}{\Lambda-\Gamma}, \tag{2.37}$$

where

$$\chi^2=\frac{m^2}{\lambda}+\frac{\gamma^2}{\lambda(\Lambda-\Gamma)}, \tag{2.38}$$

and

$$V^{(4)}_{min} = -\frac{2M^4}{\Lambda+\Gamma}-\frac{1}{\lambda}\left[m^2+\frac{\gamma^2}{\Lambda-\Gamma}\right]^2. \tag{2.39}$$

Straightforward but tedious analysis shows that in all cases there is a range of parameters such that $V^{(4)}_{min} < V^{(3)}_{min}$, $V^{(2)}_{min}$, and $v_L \ll v_R \ll \chi$.

III. MULTI-GENERATIONS; GENERAL CONSIDERATIONS

As stated in the Introduction, to go beyond the single generation case, we impose a global, axial, $U(1)_A$ symmetry at the classical level. The quantum numbers associated with this additional symmetry will distinguish the different generations[12] and provide constraints on the forms of the the fermion mass matrices.

Let there be N generations of conventional fermions $f^i_{L,R}$ accompanied by $SU(2)_L \otimes SU(2)_R$ singlet partners $F^i_{L,R}$, i=1,2,...,N and where f=u,d,ν,e, F=U,D,N,E (See Table 1). We postulate that the Lagrangian involving these fermions and the Higgs scalars is invariant under

$$f^i_{L,R} \to e^{\pm i\theta x_i} f^i_{L,R}, \quad F^i_{L,R} \to e^{\pm i\theta y_i} F^i_{L,R}, \tag{3.1}$$

where as yet unspecified $U(1)_A$ charges x_i, y_j are such that

$$x_i \neq x_j \ , \quad y_i \neq y_j \quad \text{for } i\neq j, \tag{3.2}$$

so that each generation has a distinct quantum number. We shall keep the Higgs structure to the minimum, but as well-known, the introduction of a global $U(1)_A$ symmetry necessitates a doubling of the minimum number of Higgs scalars.[11,12] A given scalar field φ and its charge-conjugate field $\tilde{\varphi}=i\tau_2\varphi^*$ are now distinguished because they have opposite $U(1)_A$ charges. When the symmetry is spontaneously broken and φ acquires a non-vanishing vacuum expectation value, φ and $\tilde{\varphi}$ generate fermion masses in different charge sectors. If φ is chosen to be the doublet $\begin{bmatrix}\varphi^+\\ \varphi^0\end{bmatrix}$, it contributes to down-charge sector and $\tilde{\varphi}$ contributes to the up-charge sector. In our model, we have four Higgs doublets

$$\varphi_L^{h_1}(1,2,1)_1 \ , \ \varphi_L(1,2,1)_1^{h_2} \ , \ \varphi_R^{-h_1}(1,1,2)_1, \ \varphi_R^{-h_2}(1,1,2)_1,$$

where h_i represent the $U(1)_A$ charges. It is easy to show that

$$h_1+h_2 = 0 \tag{3.3}$$

and therefore, the axial charges can be normalized to ±1. Under $U(1)_A$, they transform according to

$$\varphi_L^{1,-1} \to e^{\pm i\theta} \varphi_L^{1,-1}, \quad \varphi_R^{1,-1} \to e^{\pm i\theta}\varphi_R^{1,-1}. \tag{3.4}$$

The charge conjugated fields $\tilde{\varphi}^{\pm1}$ are defined by

$$\tilde{\varphi}^{\pm1} = i\tau_2(\varphi^{\mp})^*. \tag{3.5}$$

Next, let $K_{ij}=x_i+y_j$. In order to have a Yukawa-type fermion-Higgs vertex, we must have $K_{ij}=\pm1$. Table 2 summarizes the nature of the non-vanishing type of such couplings. (See also Figure 1).

Table 2

K_{ij}	Mass generating quark–Higgs couplings	
+1	$\bar{q}^i_L \varphi_L^{+1} D^j_R$	$\bar{D}^j_L \varphi_R^{+1} q^i_R$
	$\bar{q}^i_L \tilde{\varphi}_L^{+1} U^j_R$	$\bar{U}^j_L \varphi_R^{-1} q^i_R$
−1	$\bar{q}^i_L \varphi_L^{-1} D^j_R$	$\bar{D}^j_L \varphi_R^{-1} q^i_R$
	$_L^i \tilde{\varphi}_L^{-1} U^j_R$	$\bar{U}^j_L \tilde{\varphi}_R^{-1} q^i_R$

Note further that, (i) there cannot be two identical entries in the same row or column since that would violate (3.2), (ii) at least N out of N^2 K entries must equal ±1, as otherwise the associated mass submatrix ($m_{ij}=0$ if $K_{ij}\neq\pm1$) will have a vanishing determinant, leading to a zero eigenmass. We shall also ignore K–patterns which allow a larger axial symmetry and consider patterns which transform into each other by $x_i \leftrightarrow x_j$, $y_i \leftrightarrow y_j$, or $\varphi^1 \leftrightarrow \varphi^{-1}$, as equivalent.

The general form of the mass matrix is then evident. First of all, since there are no couplings in the conventional quark sectors, the mass matrices have zero entries in these sectors. The entries in the $\bar{q}^i_L \varphi_L D^j_R$ or $\bar{q}^i_L \tilde{\varphi}_L U^j_R$ sectors will involve the vacuum expectation value (VEV) $\langle\varphi^i_L\rangle = v^i_L$ and the appropriate fermion–Higgs coupling constants. Likewise, the entries in $\bar{D}^i_L \varphi^i_R q^j_R$ or $\bar{U}^i_L \tilde{\varphi}^i_R q^j_R$ will involve the VEV $\langle\varphi^i_R\rangle = v^i_R$. Finally, the $\bar{U}^i_L \sigma U^j_R$ sector, consisting of the singlet fermions, will contain the VEV $\langle\sigma\rangle = \chi$. Therefore, the mass matrix has the general form

$$M = \begin{bmatrix} 0 & M_L \\ M_R & M_X \end{bmatrix}, \qquad (3.6)$$

where M_L, M_R, and M_X are in general NxN, complex matrices. M_L (M_R) will contain the VEV's v_L^1, v_L^2 (v_R^1,v_R^2) and M_X, the VEV χ.

Following our basic assumption concerning the hierarchy in the VEVs, we assume,

$$v_L^1 \approx v_L^2 , \quad v_R^1 \approx v_R^2,$$

but

$$v_L^i << v_R^i << \chi, \quad i=2. \qquad (3.7)$$

With this assumption, we have to consider two separate cases:

1) Non-singular case; det $M_X \neq 0$. Then, the effective mass matrix for the light quarks[12] is given by M_q, where

$$M_q = -M_L M_X^{-1} M_R \qquad (3.8)$$

2) Singular case; det $M_X=0$. In this case, there is no simple expression for M_L and needs an exact treatment for the eigenvalues.

IV. PHENOMENOLOGY - THREE GENERATIONS

In this section, we shall consider in some detail the case of N=3 using the general considerations put forth in the previous section. We shall show that, in spite of the apparently large number of parameters, meaningful constraints can be imposed on them. The only significant input information we use is the values of the quark masses.

(a) $U(1)_A$ Quantum Numbers

The matrix K_{ij} consisting of the sums of the $U(1)_A$ quantum numbers has the form

$$
\begin{array}{c|cccccc}
 & x_1 & x_2 & x_3 & y_1 & y_2 & y_3 \\
\hline
x_1 & 2x_1 & x_1+x_2 & x_1+x_3 & x_1+y_1 & x_1+y_2 & x_1+y_3 \\
x_2 & x_2+x_1 & 2x_2 & x_2+x_3 & x_2+y_1 & x_2+y_2 & x_2+y_3 \\
x_3 & x_3+x_1 & x_3+x_2 & 2x_3 & x_3+y_1 & x_3+y_2 & x_3+y_3 \\
 & & & & & & \\
y_1 & y_1+x_1 & y_1+x_2 & y_1+x_3 & 2y_1 & y_1+y_2 & y_1+y_3 \\
y_2 & y_2+x_1 & y_2+x_2 & y_2+x_3 & y_2+y_1 & 2y_2 & y_2+y_3 \\
y_3 & y_3+x_1 & y_3+x_2 & y_3+x_3 & y_3+y_1 & y_3+y_2 & 2y_3
\end{array}
$$

$$
\equiv \begin{bmatrix} X & L \\ R & Y \end{bmatrix} \tag{4.1}
$$

We shall restrict our considerations to the case when the mass sub-matrices M_L and M_R in a given charge sector are Fritzsch-type, that is, in the d-sector for instance,

$$
M_L^{(d)} = \begin{bmatrix} 0 & \alpha_{12}^{(d)} v_L^1 & 0 \\ \alpha_{21}^{(d)} v_L^1 & 0 & \alpha_{23}^{(d)} v_L^2 \\ 0 & \alpha_{32}^{(d)} v_L^2 & \alpha_{33}^{(d)} v_L^1 \end{bmatrix}, \qquad M_R^{(d)} = \begin{bmatrix} 0 & \alpha_{12}^{(d)} v_R^1 & 0 \\ \alpha_{21}^{(d)} v_R^1 & 0 & \alpha_{23}^{(d)} v_R^2 \\ 0 & \alpha_{32}^{(d)} v_R^2 & \alpha_{33}^{(d)} v_R^1 \end{bmatrix}, \tag{4.2}
$$

where α_{ij} are Yukawa-type fermion-Higgs couplings and

$$
v_L^{1,2} = \langle \varphi_L^{1,2} \rangle, \; v_R^{1,2} = \langle \varphi_R^{1,2} \rangle. \tag{4.3}
$$

This implies that the sub-matrices L and R of K have a prescribed form, namely,

$$
L = \begin{bmatrix} 0 & h_1 & 0 \\ h_1 & 0 & h_2 \\ 0 & h_2 & h_1 \end{bmatrix}, \quad R = \begin{bmatrix} 0 & -h_2 & 0 \\ -h_2 & 0 & -h_1 \\ 0 & -h_1 & -h_2 \end{bmatrix}. \tag{4.4}
$$

From (4.1) and (4.4),

$$x_1+y_2=x_2+y_1=h_1, \qquad x_2+y_3=x_3+y_2=h_2, \qquad x_3+y_3=h_1, \tag{4.5}$$

we find,

$$\begin{aligned} &x_1=x, && y_1=3h_1-2h_2-x \\ &x_2=2h_2-2h_1+x, && y_2=h_1-x \\ &x_3=h_2-h_1+x && y_3=2h_1-h_2-x, \end{aligned} \tag{4.6}$$

where x is an arbitrary integer and $h_1=-h_2=1$. The full K-matrix then takes the form,

$$K = \left[\begin{array}{ccc|ccc} 2x & 2x-4 & 2x-2 & 5 & 1 & 3 \\ 2x-4 & 2x-8 & 2x-6 & 1 & -3 & -1 \\ 2x-2 & 2x-6 & 2x-4 & 3 & -1 & 1 \\ \hline 5 & 1 & 3 & 10-2x & 6-2x & 8-2x \\ 1 & -3 & -1 & 6-2x & 2-2x & 4-2x \\ 3 & -1 & 1 & 8-2x & 4-2x & 6-2x \end{array}\right]. \tag{4.7}$$

Since in order that the fermion Higgs couplings exist, we need $K_{ij}=\pm1$, we obtain the desired form for L and R. X cannot have ±1 in any of its entry as to be expected from the fact that the assumed properties of the Higgs fields do not permit a $\bar{q}_L q_R \varphi$ vertex.

Now, in order to have non-vanishing entries in Y, we should have zeroes in Y. x has four possibilities, x=2, x=3, x=4, or x=5, out of which only x=3 gives rise to a non-singular form for the corresponding part of the mass matrix. We shall consider this case first.

b) The Nonsingular Case

With x=3,

$$K = \left[\begin{array}{ccc|ccc} 6 & 2 & 4 & 5 & 1 & 3 \\ 2 & -2 & 0 & 1 & -3 & -1 \\ 4 & 0 & 2 & 3 & -1 & 1 \\ \hline 5 & 1 & 3 & 4 & 0 & 2 \\ 1 & -3 & -1 & 0 & -4 & -2 \\ 3 & -1 & 1 & 2 & -2 & 0 \end{array}\right], \tag{4.8}$$

from which we can read of the non-vanishing elements of the mass matrices. As already noted, the top-left part of the 3x3 matrix will be zero. The sub-mass matrices in the two charge sectors are given by,

$$M_L^d = \begin{bmatrix} 0 & \alpha_{12}^d v_L^1 & 0 \\ \alpha_{21}^d v_L^1 & 0 & \alpha_{23}^d v_L^2 \\ 0 & \alpha_{32}^d v_L^2 & \alpha_{23}^d v_L^1 \end{bmatrix}, \quad M_R^d = \begin{bmatrix} 0 & \alpha_{12}^d v_R^1 & 0 \\ \alpha_{21}^d v_R^1 & 0 & \alpha_{23}^d v_R^2 \\ 0 & \alpha_{32}^d v_R^2 & \alpha_{33}^d v_R^1 \end{bmatrix}, \tag{4.9}$$

$$M_L^u = \begin{bmatrix} 0 & \alpha_{12}^u v_L^{2*} & 0 \\ \alpha_{21}^u v_L^{2*} & & \alpha_{23}^u v_L^{1*} \\ 0 & \alpha_{32}^u v_L^{1*} & \alpha_{33}^u v_L^{2*} \end{bmatrix}, \quad M_R^u = \begin{bmatrix} 0 & \alpha_{12}^u v_R^{2*} & 0 \\ \alpha_{21}^u v_R^{2*} & 0 & \alpha_{23}^u v_R^{1*} \\ 0 & \alpha_{32}^u v_R^{1*} & \alpha_{33}^u v_R^{2*} \end{bmatrix}, \tag{4.10}$$

and

$$M_\sigma^d = \begin{bmatrix} 0 & \gamma_{12}^d \chi & 0 \\ \gamma_{21}^d \chi & 0 & 0 \\ 0 & 0 & \gamma_{33}^d \chi \end{bmatrix}, \quad M_\sigma^u = \begin{bmatrix} 0 & \gamma_{12}^u \chi & 0 \\ \gamma_{21}^u \chi & 0 & 0 \\ 0 & 0 & \gamma_{33}^u \chi \end{bmatrix}. \tag{4.11}$$

Equations (4.9), (4.10), and (4.11) exhibit the correlations between the down- and up-charge sectors ($v_L^1 \leftrightarrow v_L^{2*}$, $v_R^1 \leftrightarrow v_R^{2*}$) and present the most general form within the framework of our model. We assume further that

a) The Y-couplings are real and symmetric,

b) $\gamma_{12}\chi = \gamma_{33}\chi = \chi$,

c) the VEVs satisfy

$$v_L^i \ll v_R^i \ll \chi \ , \ i=1,2, \tag{4.12}$$

and

$$|v_L^1| \approx |v_L^2| = v_L \qquad |v_R^1| \approx |v_R^2| = v_R \tag{4.13}$$

For calculational simplicity, we shall take them to be equal. We shall neglect at the moment their phases and assume essentially that v_L, v_R are real. Hence, there will be no CP-violation in the gauge sector of our model.

With the above assumptions and according to the discussion in the previous section, the mass matrix M_q for light quarks has the generic form

$$M_q = -M_L M_\sigma^{-1} M_R,$$

$$= \begin{bmatrix} 0 & x & 0 \\ x & z & y \\ 0 & y & w \end{bmatrix}, \qquad (4.14)$$

where for the d-sector,

$$x = (\alpha_{12}^d)^2 \frac{v_L v_R}{\chi}, \quad z = (\alpha_{23}^d)^2 \frac{v_L v_R}{\chi}, \quad w = (\alpha_{33}^d)^2 \frac{v_L v_R}{\chi},$$

and

$$y = \frac{v_L v_R}{\chi}(\alpha_{12}^d \alpha_{23}^d + \alpha_{23}^d \alpha_{33}^d), \qquad (4.15)$$

with corresponding expressions for x, y, z, w for the u-secotr in which $\alpha_{ij}^d \to \alpha_{ij}^u$.

If λ_1, λ_2, λ_3 are the eigenvalues of the real, symmetric mass matrix (4.14),

$$\lambda_1+\lambda_2+\lambda_3 = z+w$$

$$\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1 = -x^2+zw-y^2,$$

$$\lambda_1\lambda_2\lambda_3 = -x^2 w, \qquad (4.16)$$

which lead to the solution

$$z = (\lambda_1+\lambda_2+\lambda_3)-w,$$

$$x^2 = -\frac{\lambda_1\lambda_2\lambda_3}{w},$$

$$wy^2 = (\lambda_1-w)(\lambda_2-w)(\lambda_3-w), \qquad (4.17)$$

in terms of the eigenvalues λ_1, λ_2, λ_3 and w. The positivity of x, z, w requires that one of the eigenvalues be negative. We shall assume λ_1 to be negative and identify

$$\lambda_1=-m_1,\ \lambda_2=m_2,\ \lambda_3=m_3, \tag{4.18}$$

where m_1, m_2, m_3 are the quark masses in the appropriate sectors. Hence,

$$\begin{aligned} z&=(-m_1+m_2+m_3)-w \\ x^2&=\frac{m_1m_2m_3}{w}, \\ wy^2 &= (w+m_1)(w-m_2)(m_3-w). \end{aligned} \tag{4.19}$$

It is straightforward to compute the eigenvectors and the real, orthogonal matrix that diagonalizes M_q in terms of m_1, m_2, m_3, and w. Thus, if

$$OM_qO^T = \mathrm{diag}(-m_1,\ m_2,\ m_3),$$

$$O = \begin{bmatrix} \left[\frac{m_2m_3(w+m_1)}{w(m_2+m_1)(m_3-m_1)}\right]^{1/2} & \left[\frac{m_1m_3(w-m_2)}{w(m_1+m_2)(m_3-m_2)}\right]^{1/2} & \left[\frac{m_1m_2(m_3-w)}{w(m_3-m_2)(m_3+m_1)}\right]^{1/2} \\ -\left[\frac{m_1(w+m_1)}{(m_1+m_2)(m_3-m_1)}\right]^{1/2} & \left[\frac{m_2(w-m_2)}{(m_2+m_1)(m_3-m_2)}\right]^{1/2} & \left[\frac{m_3(m_3-w)}{(m_3-m_2)(m_3+m_1)}\right]^{1/2} \\ \left[\frac{m_1(w-m_2)(m_3-w)}{w(m_2+m_1)(m_3+m_1)}\right]^{1/2} & -\left[\frac{m_2(w+m_1)(m_3-w)}{w(m_2+m_1)(m_3-m_2)}\right]^{1/2} & \left[\frac{m_3(w+m_1)(w-m_2)}{w(m_3-m_2)(m_3+m_1)}\right]^{1/2} \end{bmatrix}. \tag{4.20}$$

If we consider the quark mass parameters[13] currently believed to be established, we can expand the above matrix and to a very good approximation retain only the leading terms. Then O is given by

$$O = \begin{bmatrix} 1 & \left[\frac{m_1}{m_2}\right]^{1/2} & \left[\frac{m_1m_2}{wm_3}\left(1-\frac{w}{m_3}\right)\right]^{1/2} \\ -\left[\frac{m_1w}{m_2m_3}\right]^{1/2} & \left[\frac{w}{m_3}\right]^{1/2} & \left[1-\frac{w}{m_3}\right]^{1/2} \\ \left[\frac{m_1}{m_2}\left(1-\frac{w}{m_3}\right)\right]^{1/2} & -\left[1-\frac{w}{m_3}\right]^{1/2} & \left[\frac{w}{m_3}\right]^{1/2} \end{bmatrix} \tag{4.21}$$

The Cabibbo–Kobayashi–Maskawa matrix V_{CKM} is then given by

$$[V_{CKM}] = O^{(u)} O^{(d)T}, \tag{4.22}$$

where $O^{(u)}$ and $O^{(d)T}$ are obtained by substituting the mass parameters in the u– and d–sectors respectively in (4.22). For simplicity, we shall use d, s, b,..., to represent the corresponding masses m_d, m_s, m_b...,. Then,

$$\begin{aligned}[V_{CKM}]_{12} &= -\left(\frac{w_d}{b}\right)^{1/2}\left(\sqrt{\frac{d}{s}} - \sqrt{\frac{u}{c}}\right) \\ &\quad + \left[\frac{u}{t}\frac{c}{w_u}\left(1-\frac{w_u}{t}\right)\left(1-\frac{w_d}{b}\right)\right]^{1/2} \\ &\approx -\left(\frac{w_d}{b}\right)^{1/2}\left(\sqrt{\frac{d}{s}} - \sqrt{\frac{u}{c}}\right)\end{aligned} \tag{4.23}$$

$$\begin{aligned}[V_{CKM}]_{13} &= \left(1-\frac{w_d}{b}\right)^{1/2}\left(\sqrt{\frac{d}{s}} - \sqrt{\frac{u}{c}}\right) \\ &\quad + \left[\frac{u}{t}\frac{c}{w_u}\left(1-\frac{w_u}{t}\right)\frac{w_d}{b}\right]^{1/2} \\ &\approx \left(1-\frac{w_d}{b}\right)^{1/2}\left(\sqrt{\frac{d}{s}} - \sqrt{\frac{u}{c}}\right),\end{aligned} \tag{4.24}$$

$$[V_{CKM}]_{23} = -\left[\left(1-\frac{w_d}{b}\right)\frac{w_u}{t}\right]^{1/2}\left(1+\sqrt{\frac{ud}{cs}}\right) + \left[\left(1-\frac{w_u}{t}\right)\left(\frac{w_d}{b}\right)\right]^{1/2} \tag{4.25}$$

$$\begin{aligned}[V_{CKM}]_{31} &= -\left(1-\frac{w_u}{t}\right)^{1/2}\left(\sqrt{\frac{d}{s}} - \sqrt{\frac{u}{c}}\right) \\ &\quad + \left[\frac{d}{b}\frac{s}{w_d}\left(1-\frac{w_d}{b}\right)\frac{w_u}{t}\right]^{1/2} \\ &\approx -\left(1-\frac{w_u}{t}\right)^{1/2}\left(\sqrt{\frac{d}{s}} - \sqrt{\frac{u}{c}}\right).\end{aligned} \tag{4.26}$$

Eqs. (4.23), (4.24). and (4.25) express the experimentally measured quantities V_{us}, V_{ub}, and V_{cb} in terms of the quark mass parameters and the two scale parameters w_d and w_u. Out of the three quantities, only V_{us} which is related to the well-known Cabibbo angle, is well established experimentally. The other two are not well established owing to the uncertainties in the measurements as well as the theoretical model dependence in the interpretation of the data. The most recent Particle Data Book gives the following ranges for the values of V_{us}, V_{ub}, and V_{cb}:

V_{us}: .217 to .223, V_{ub}: 0.003 to 0.010, V_{cb}: 0.03 to 0.062. These uncertainties coupled with the uncertainties in the values of the quark mass parameters (especially the light quarks) make a precise quantitative comparison of (4.23)-(4.25) extremely difficult at the present stage. We look only for qualitative and semi-quantitative agreement.

First of all, it is gratifying to note that the relation (4.23) for the Cabibbo angle is a consequence of the model. Indeed, in the case of two generations, $\sin\theta_c = (\sqrt{\frac{\bar{d}}{s}} - \sqrt{\frac{\bar{u}}{c}})$ provides an extremely good relation consistent with experimental information. This suggests that in the case of three generations, the factor (w_d/b) in (4.23) should be close to unity, which implies from (4.24) that V_{ub} is naturally suppressed.[14] The dominant term in (4.25) is the second factor which can be fit to V_{cb} by choosing an appropriate value for (w_u/t). Then, (4.26) predicts V_{td} for which, at present, there is only indirect information.[15] But it is expected to be large, of the same order of magnitude as V_{cb}. That is indeed the case, as one can see from a comparison of (4.25) with (4.26).

With the range of values for the mass parameters, the factor $(\sqrt{\frac{\bar{d}}{s}} - \sqrt{\frac{\bar{u}}{c}})$ varies from .1148 to .2382. This large variation is due to large uncertainties in d, s, and u quark mass values. Taking this variation into account, we can fit for instance, to

$$V_{us} = .22,\ V_{cb} = .05,\ V_{ub} = .007, \tag{4.27}$$
by choosing
$$\left(\sqrt{\frac{\bar{d}}{s}} - \sqrt{\frac{\bar{u}}{c}}\right) = .2211,\quad \frac{W_d}{b} = .9995,\quad \frac{W_u}{t} = .9948. \tag{4.28}$$
We then predict $V_{td} \approx 0.016$ from (4.26).

Further, from (4.15),
$$\frac{x}{w} = \left(\frac{\alpha_{12}}{\alpha_{33}}\right)^2 = \left(\frac{m_1 m_2 m_3}{W^3}\right)^{1/2} \tag{4.29}$$
and
$$\frac{z}{w} = \left(\frac{\alpha_{23}}{\alpha_{33}}\right)^2 = \frac{m_3 + m_2 - m_1 - W}{W}\ . \tag{4.30}$$
In the d–sector, using the values for mass parameters and $W_d \approx m_b$ (from (4.28)), we find
$$\alpha^d_{12}/\alpha^d_{33} = \sqrt[4]{\frac{ds}{b^2}} \simeq .16 \text{ to } .25, \tag{4.31}$$
$$\alpha^d_{23}/\alpha^d_{33} \approx \sqrt{\frac{s}{b}} \simeq .15 \text{ to } .21, \tag{4.32}$$
which shows that the ratios of Y–couplings are quite reasonable, in the sense that they do not exhibit the same degree of hierarchy as the masses. The latter would be the case in a straightforward extension of the standard model to include more generations.

Going a step further, if we use again the value $v_L = 246$ GeV and $v_R/\chi = 10^{-6}$, we are led to absolute values of the couplings
$$|\alpha_{12}|=30,\ |\alpha_{23}|=24.5,\ \text{and}\ |\alpha_{33}|=150. \tag{4.33}$$
These representative values for Y–couplings are all within one order of magnitude.

Thus, the simplified version of the model provides a satisfactory extension to three generations of quarks. Although the quark masses are not predicted, the matrix elements V_{us}, V_{cb}, V_{ub} and hence the CKM mixing angles, and also the relevant Y–couplings are expressed in terms of the quark masses.

c) The Singular Case

We consider briefly one singular case by choosing x=4 in (4.7). $M_{\sigma}^{d,u}$ is then singular. There is no approximate form for the light quark mass matrix in this case. The eigenvalue equation is given by

$$\lambda^6 - \lambda^4[\chi^2 + 3V_LV_R] - 2\lambda^3[V_LV_R\chi]$$

$$+ 2\lambda^2[V_LV_R\chi^2 + V_L^2V_R^2] + 4\lambda\, V_L^2V_R^2\chi - V_L^3V_R^3 = 0. \tag{4.34}$$

Under the assumed hierarchy, we can show that the approximate eigenvalues fall into three categories

$$\text{Light:} \quad \lambda_{1,2} \pm \frac{V_LV_R}{\sqrt{2}\chi},$$

$$\text{Intermediate:} \quad \lambda_{3,4} = \pm\sqrt{2V_LV_R},$$

$$\text{Heavy:} \quad \lambda_{5,6} = \pm\chi. \tag{4.35}$$

$\lambda_{1,2}$, being the smallest, can be identified with the light quarks of the first two generations. One of λ_3, λ_4 can represent the heavier third generation quark with one of the heavy singlet quark being almost degenerate with it. $\lambda_{5,6}$ represent the remaining singlets.

The predicted spectrum has indeed features corresponding to reality. The quarks of the first two generations in the light of the heavy χ are near degenerate. The quark belonging to the third generation is much heavier (corresponding to the fact that b and t quarks are much heavier than their corresponding partners in the first two generations). The spectrum also predicts a heavy singlet in the mass range of the third generation quark.

V. SUMMARY AND CONCLUSIONS

We have explored in this paper the consequences of a model based on the idea of a Universal Seesaw Mechanism for explaining the mass hierarchies and the mixing angles in the quark sector. The basic idea is the generalization of the seesaw mechanism to generate superlight neutrino by postulating a heavy right-handed Majorana neutrino and a Majorana mass term. Likewise, within the framework of left-right symmetry, if every fermion is accompanied by a heavy left-right singlet fermion, the mass matrix for every fermion assumes a seesaw form and gives rise to the possibility of light mass quarks and leptons.

In the case of a single generation, the hierarchy assumption $v_L \ll v_R \ll \chi$ leads to light quark masses and light mass charged lepton. At the same time, without any further assumption, one is lead to a superlight neutrino in a natural way. These results found earlier are further strengthened in this investigation by 1) the demonstration that the assumed hierarchy can be a consequence of spontaneous left-right symmetry breaking caused by an odd parity left-right symmetric singlet, and 2) the fact that no additional hierarchy assumption in the Yukawa-type couplings between the fermions and the Higgs scalars needs to be invoked. These nice results prompt one to consider the realistic case of three generations. By postulating a global, axial U(1) symmetry, we distinguish the different generations and study the constraints imposed by the additional symmetry on the form of the mass matrices. While in general several forms of mass matrices are allowed, we confine our attention to the so-called Fritzsch form for the submatrices. We ignore in this investigation, CP violation by assuming that all the VEVs and the Y-couplings are real, so that the Kobayashi-Maskawa phase is zero. This is only for the sake of simplicity and to test the viability of USM in providing a reasonably satisfactory answer for the known Cabibbo-Kobayashi-Maskawa matrix elements.

The results of the simplified version of the model are indeed encouraging. If the quark masses are assumed to be known, the results depend on two new parameters, one in each charge sector of the quarks. The known experimental information concerning the matrix elements V_{us}, V_{ub}, and V_{cb} place stringent conditions on these two parameters. There is an inner consistency in the correlations of the theoretical expressions for these matrix elements with their observed values. Further, as in the single generation case, the Y–couplings do not exhibit the same degree of hierarchy as the mass parameters. Thus on the whole the model provides a promosing approach to the mass hierarchy and the phenomenology of generation mixing interactions.

It must be said, however, that the model even within its own framework has considerable flexibility and several parameters. The present investigation has concentrated on reducing the dependence on the arbitrary parameters and seeking combination of parameters which are directly related to observed quantities.[16] It should also be noted that at present there are too many uncertainties in our experimental knowledge. As a result, a detailed numerical confrontation of our model (or any other model) makes it extremely difficult, perhaps even pointless. We have to wait for the discovery of the top quark and its decay characteristics, and more precise and independent measurements of all the CKM matrix elements for such a confrontation.

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13. We use the following values for the mass parameters:

$m_u = 5.1 \pm 1.5$ MeV	$m_c = 8.9 \pm 1.6$ MeV
$m_c = 1.35 \pm 0.05$ GeV	$m_s = 0.175 \pm 0.055$ GeV
$m_t \geq 100$ GeV	$m_b = 5.3 \pm 0.1$ GeV

The above values are at the mass scale of 1 GeV. The top quark mass value implies that its physical mass is $\geq$ 60 GeV. For details see, J. Gasser and H. Leutwyler, Physics Reports 87, No. 3, 1982.

14. In our model V_{ub} has a lower limit set by the condition $m_d < m_2+m_3-m_1$ so that $z>0$.

15. H. Harari, in the Proceedings of CP Violation in Particle Physics and Astrophysics, Chateau de Blois, France May 22-26, 1989, (to be published). Also see G. Altarelli, in the proceedings of the 1989 International Symposium on Heavy Quark Physics, Ithaca, NY, June 13-17, 1989, also to be published.

16. The general idea here is to seek expressions for the mixing angles and Yukawa-type couplings in terms of quark mass ratios which are better determined than the masses of individual quarks. Our model is not unique in this respect. For other examples, see for instance, Z.G. Berezhiani, Phys. Lett. B129, 99 (1983); Phys. Lett. B150, 177 (1985); F. del Augila, G.L. Kane, and M. Quros, Phys. Lett. B196, 531 (1987); M. Gronau, R. Johnson, and J. Schechter, Phys. Rev. D39, 1913 (1989). For a review and extensive lists of references, see Kyungsik Kang, Physics of Flavor Mixing, Brown-HET-690 (unpublished).

Table 2

		$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$	$U(1)_A$
Conventional quarks	$\begin{pmatrix} u \\ d \end{pmatrix}^i_L$	3	2	1	1/3	x_i
	$\begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}^i_L$	3^*	1	2	−1/3	x_i
Heavy 'Singlet' quarks	U^i_L	3	1	1	4/3	y_i
	D^i_L	3	1	1	−2/3	y_i
	$\bar{U}^i_L$	3^*	1	1	−4/3	y_i
	$\bar{D}^i_L$	3^*	1	1	2/3	y_i
Higgses	$\varphi^{1,2}_L$	1	2	1	−1	±1
	$\varphi^{1,2}_R$	1	1	2	−1	∓1
	σ	1	1	1	0	0

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